Chess and Mathematics in UK Secondary Schools

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Maths in UK Schools – KS1 (up to 7 years)

- Numbers: 5 + 7; 2 x 4, 7 – 4
- Patterns: A A B A A B A A ?
- Shapes: squares, circles, rectangles
- Lots of overlap with introducing chess board and pieces

Maths in UK Schools – KS2 (7-11 years)

- Numbers: 14+ 27; 7 x 6, 27/4
- Patterns: 3, 5, 7, 9, 11, ??
- Shapes: more advanced shapes, transformations
- Data – finding averages
- Some overlap with the chess board and pieces

Maths in UK Schools – KS3 (11 -14 years)

- Numbers: Prime numbers, highest common factor
- Patterns: 3, 5, 7, 9, 11 … nth term?
- Shapes: angles in regular polygons
- Data – pie charts, scatter diagrams
- Algebra – solving equations
- Little overlap with the chess board and pieces

Maths in UK Schools – KS4 (14 - 16 years)

- Solving quadratic equation
- Cumulative frequency diagram
- Congruent triangles
- Very little overlap with the chess board and pieces

Maths in UK Schools – KS5 (16 - 18 years)

- Calculus – differentiation and integration
- Trigonometry
- Quadratic simultaneous equations
- Very little overlap with the chess board and pieces

Inverting 3x3 Matrices
(Upper sixth, 17 years, Further Maths)

Consider the matrix $$\mathbf{M} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

First we need to find the cofactors. These are found by crossing out the row/column of an entry and calculating the 2x2 determinant created, and then finding the correct sign. For instance $\mathbf{A}_1$ the cofactor of a_1 is

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \qquad \mathbf{A}_1 = \det\begin{pmatrix} b_2 & c_2 \\ b_3 & c_3 \end{pmatrix}$$

Inverting 3x3 Matrices

The signs used are in a chess board pattern:

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

Thus $\mathbf{A}_2$ the cofactor of a_2 is

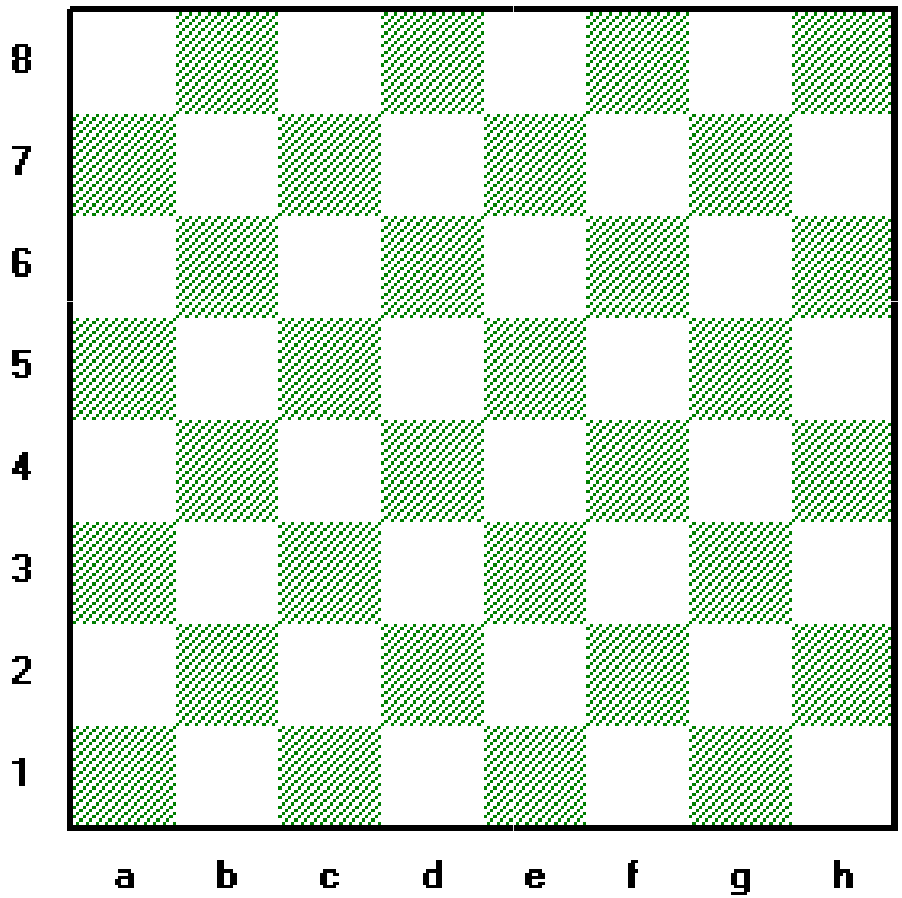
$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

$$\mathbf{A}_2 = -\det\begin{pmatrix} b_1 & c_1 \\ b_3 & c_3 \end{pmatrix}$$

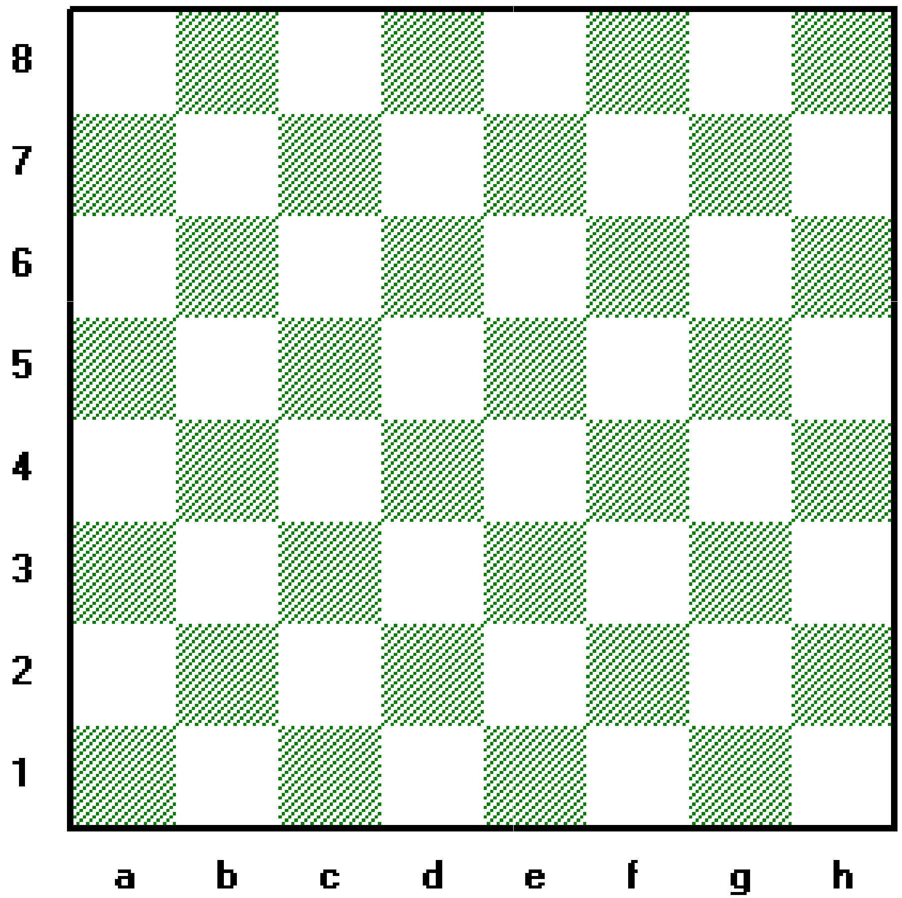
$$\mathbf{A}_3 = \det\begin{pmatrix} b_1 & c_1 \\ b_2 & c_2 \end{pmatrix} \qquad \mathbf{B}_1 = -\det\begin{pmatrix} a_2 & c_2 \\ a_3 & c_3 \end{pmatrix}$$

Etc.

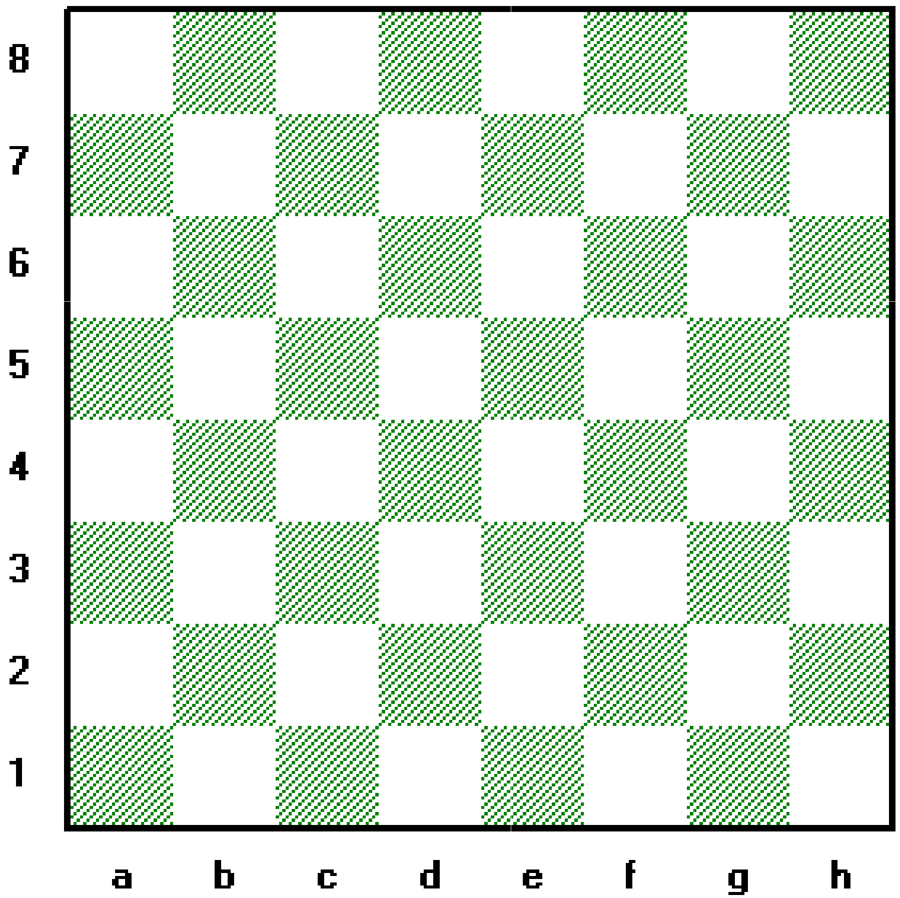
Chess Board Puzzle –
used in year 9 Royal Institution Maths Masterclass



Chess Board Puzzle –
Also used in sixth form (16 years) as a simple example of a Geometric sequence

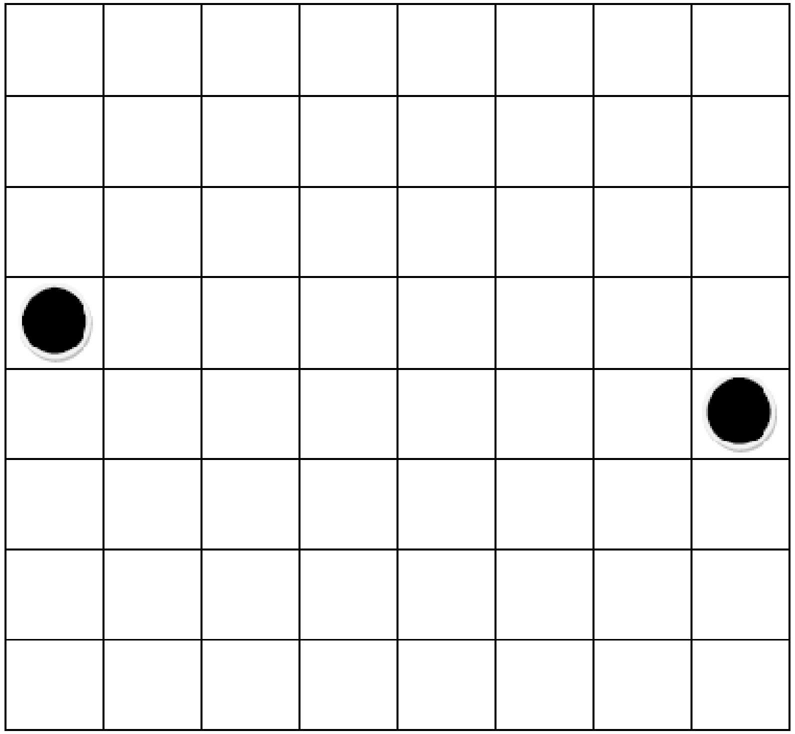


8 queens problem – recreational maths



UKMT Team Maths Challenge 2011
National Final

Station 6 Worksheet



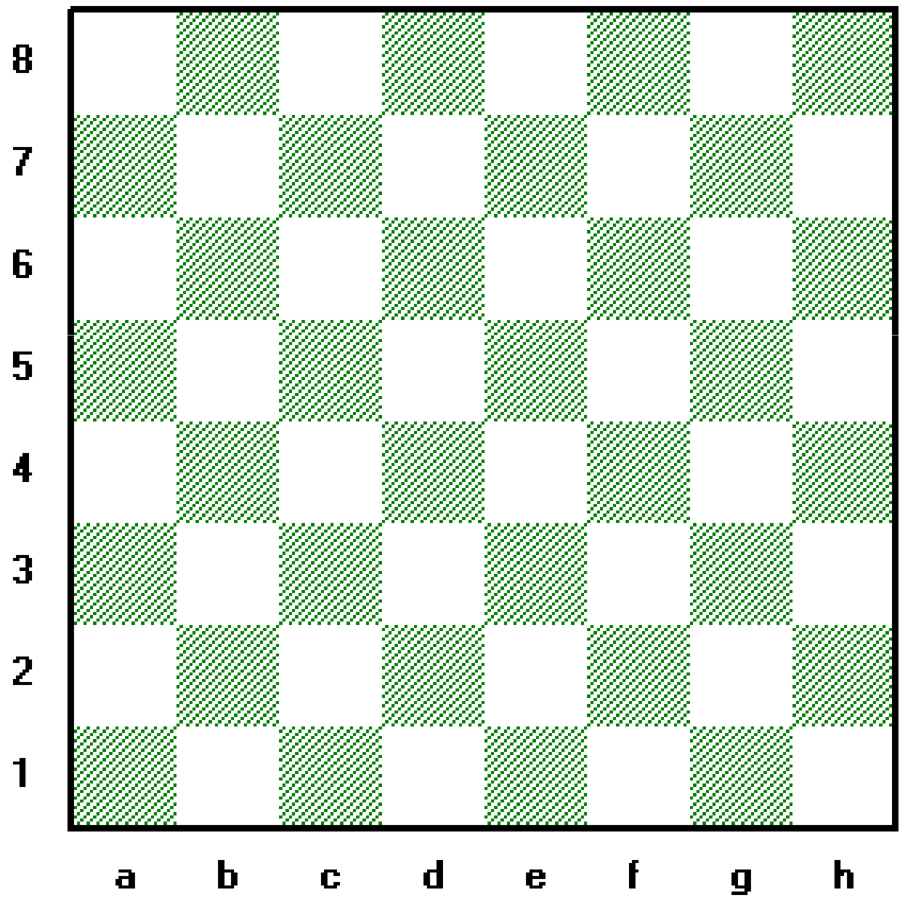
Station 6

You are given an 8 by 8 grid with the positions of two counters already placed.

You have to place a further six counters on the grid so that:

- there is no more than one counter in any column, row or diagonal (long or short);
- the final pattern of counters exhibits rotational symmetry of order 2.

2008/9 British Mathematical Olympiad Round 1: Thursday, 4 December 2008

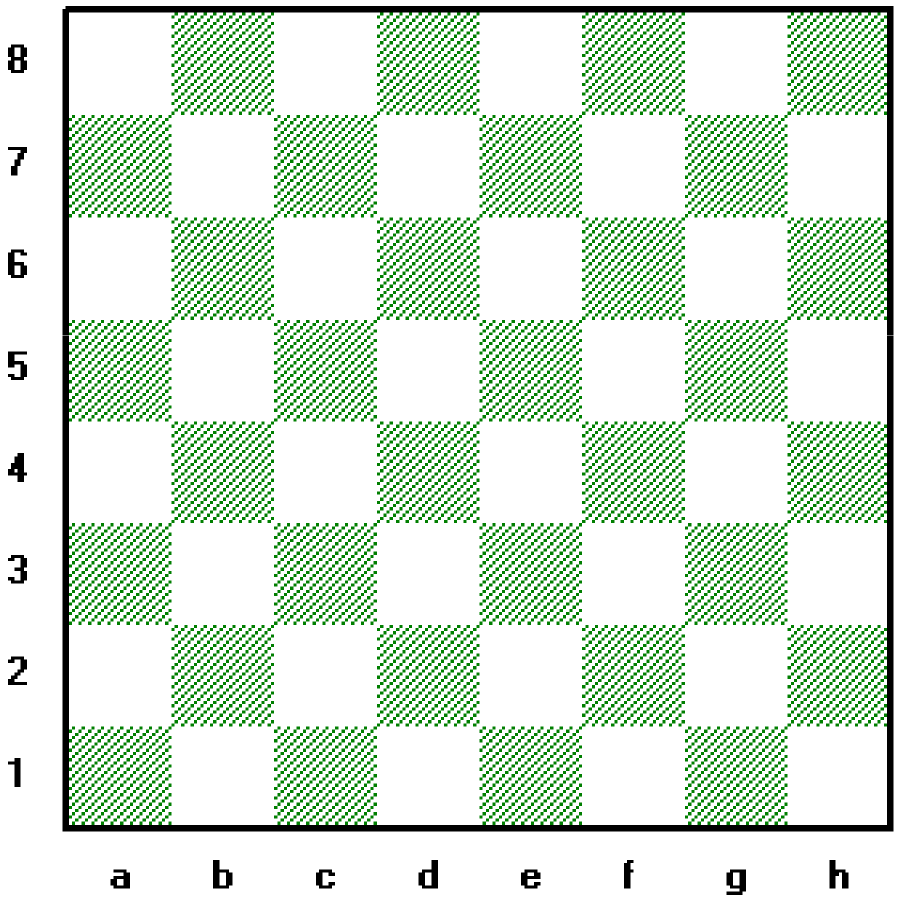


Consider a standard 8 × 8 chessboard consisting of 64 small squares coloured in the usual pattern, so 32 are black and 32 ar white.

A zig-zag path across the board is a collection of eight white squares, one in each row, which meet at their corners.

How many zig-zag paths are there?

2010/11 British Mathematical Olympiad
Round 1: Thursday, 2 December 2010

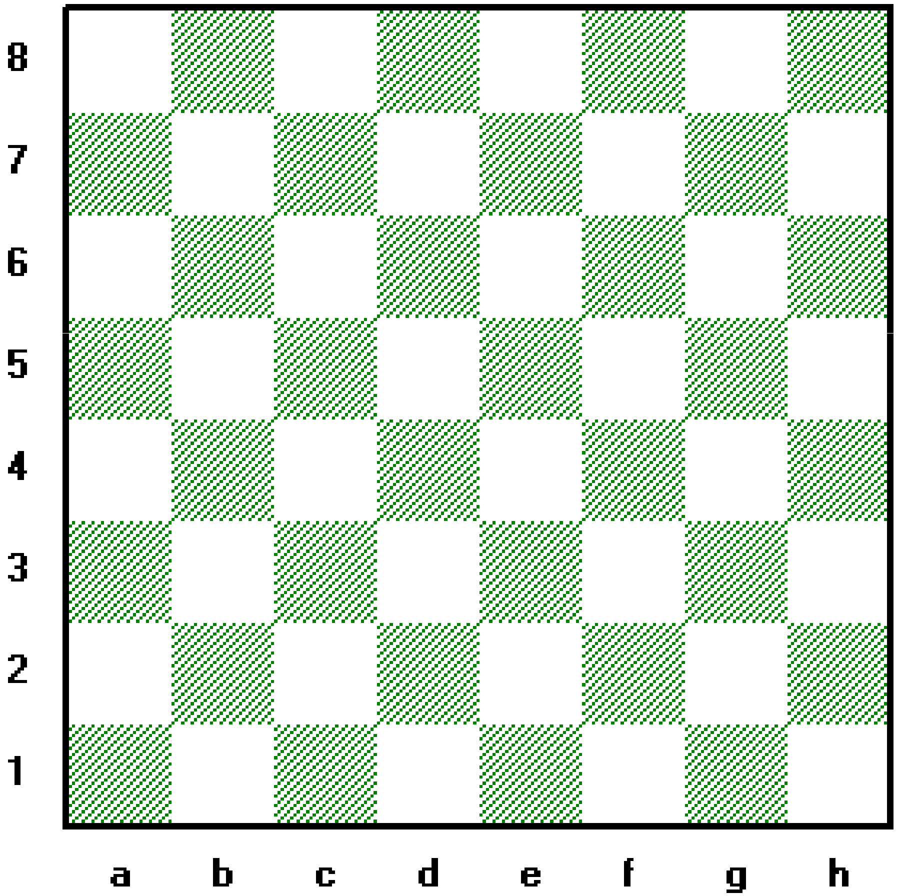


Isaac has a large supply of counters, and places one in each of the 1×1 squares of an 8×8 chessboard. Each counter is either red, white or blue.

A particular pattern of coloured counters is called an arrangement. Determine whether there are more arrangements which contain an even number of red counters or more arrangements which contain an odd number of red counters.

Note that 0 is an even number.

2012/13 British Mathematical Olympiad Round 1: Friday, 30 November 2012



Isaac places some counters onto the squares of an 8 by 8 chessboard so that there is at most one counter in each of the 64 squares.

Determine, with justification, the maximum number that he can place without having five or more counters in the same row, or in the same column, or on either of the two long diagonals.

UKMT Team Maths Challenge 2006
Poster Competition
Circuits and Tours—questions

A rook is a chess piece which moves in a straight line on an $m \times n$ rectangular board any number of squares along a row or a column.

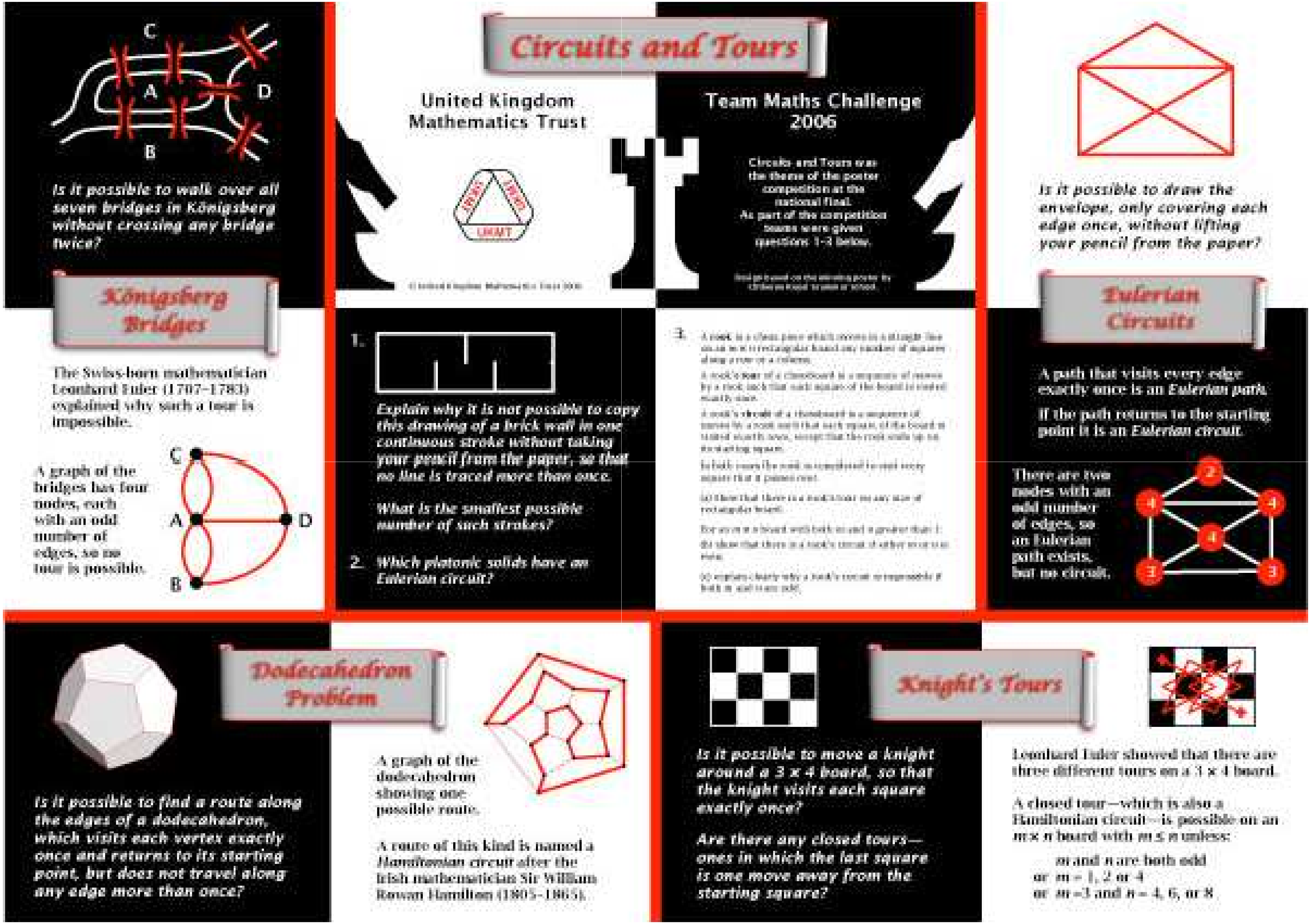
A rook's tour of a chessboard is a sequence of moves by a rook such that each square of the board is visited exactly once.

A rook's circuit of a chessboard is a sequence of moves by a rook such that each square of the board is visited exactly once, except that the rook ends up on its starting square.

In both cases the rook is considered to visit every square that it passes over.

(a) Show that there is a rook's tour on any size of rectangular board.

For an $m \times n$ board with both m and n greater than 1:
(b) show that there is a rook's circuit if either m or n is even;
(c) explain clearly why a rook's circuit is impossible if both m and n are odd.


Circuits and Tours
United Kingdom Mathematics Trust
Team Maths Challenge 2006
Circuits and Tours was the theme of the poster competition at the national final. As part of the competition teams were given questions 1–3 below.
C
A
D
B
Is it possible to walk over all seven bridges in Königsberg without crossing any bridge twice?
Königsberg Bridges
The Swiss-born mathematician Leonhard Euler (1707–1783) explained why such a tour is impossible.
A graph of the bridges has four nodes, each with an odd number of edges, so no tour is possible.
C
A
D
B
1.
Explain why it is not possible to copy this drawing of a brick wall in one continuous stroke without taking your pencil from the paper, so that no line is traced more than once.
What is the smallest possible number of such strokes?
2. Which platonic solids have an Eulerian circuit?
3.
Is it possible to draw the envelope, only covering each edge once, without lifting your pencil from the paper?
Eulerian Circuits
A path that visits every edge exactly once is an Eulerian path.
If the path returns to the starting point it is an Eulerian circuit.
There are two nodes with an odd number of edges, so an Eulerian path exists, but no circuit.
2
4
4
4
3
3
Dodecahedron Problem
Is it possible to find a route along the edges of a dodecahedron, which visits each vertex exactly once and returns to its starting point, but does not travel along any edge more than once?
A graph of the dodecahedron showing one possible route.
A route of this kind is named a Hamiltonian circuit after the Irish mathematician Sir William Rowan Hamilton (1805–1865).
Knight's Tours
Is it possible to move a knight around a 3 × 4 board, so that the knight visits each square exactly once?
Are there any closed tours—ones in which the last square is one move away from the starting square?
Leonhard Euler showed that there are three different tours on a 3 × 4 board.
A closed tour—which is also a Hamiltonian circuit—is possible on an m × n board with m ≤ n unless:
m and n are both odd
or m = 1, 2 or 4
or m = 3 and n = 4, 6, or 8

Use of Chess in Maths in secondary school

- Very little used
- Used for some interesting recreational maths puzzles
- Even there the main link is the chess board not chess playing.
- So chess is not of great use in maths education
- However

Chess and Maths in secondary school

- Most good school chess players are strong in maths
- Many people running school chess teams are maths teachers
- Many people running school chess teams also run the school maths teams
- What is the link ??

Chess and Maths in secondary school

- What is the link between chess and maths?
- One part is that both require abstract thought.