



Latin Erdős

Jorge Nuno Silva

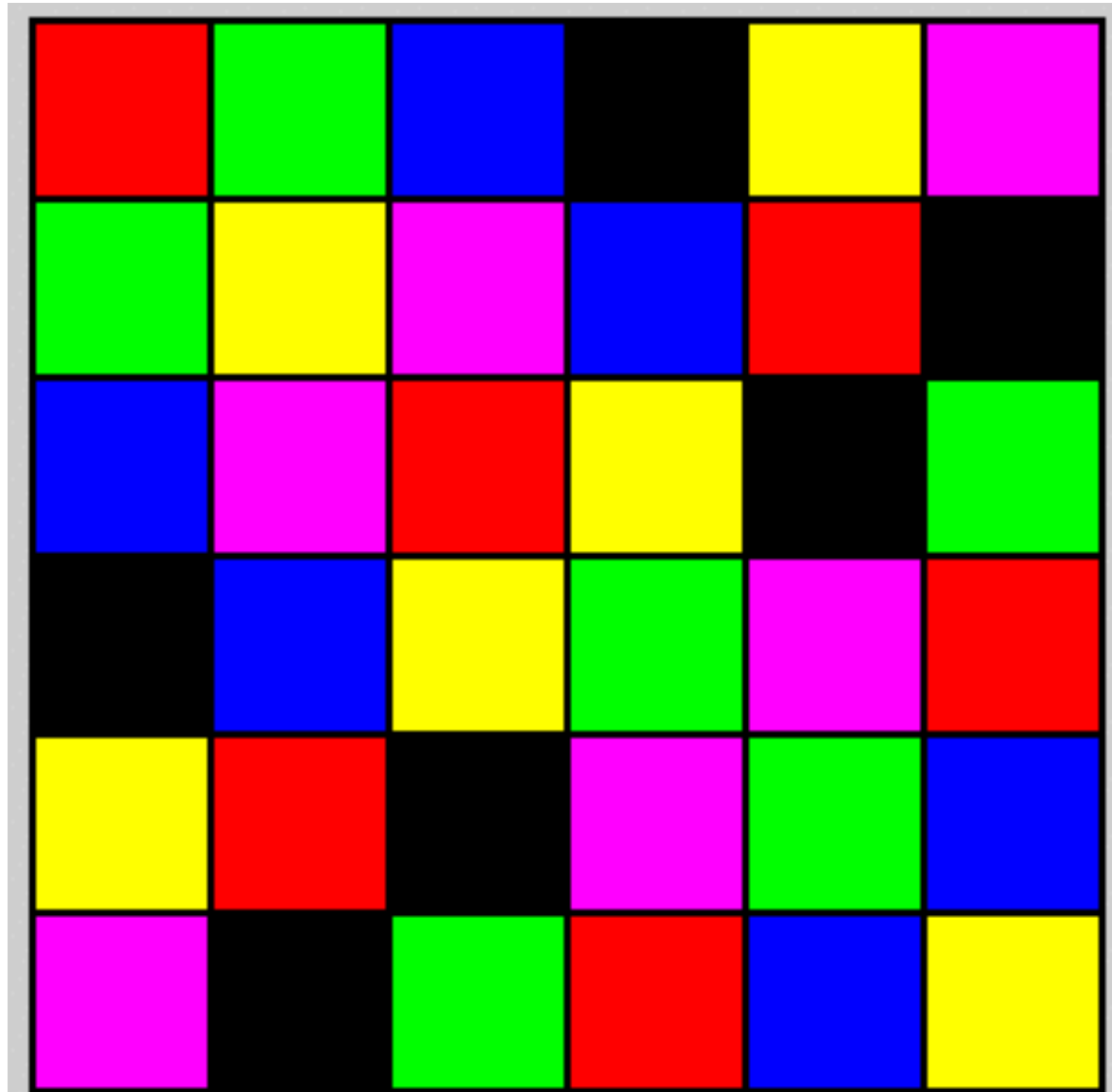
University of Lisbon-CIUHCT

Chess and Mathematics

London Conference 6-7 December



Latin Squares

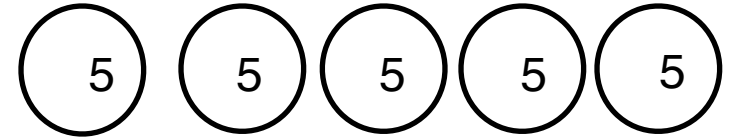
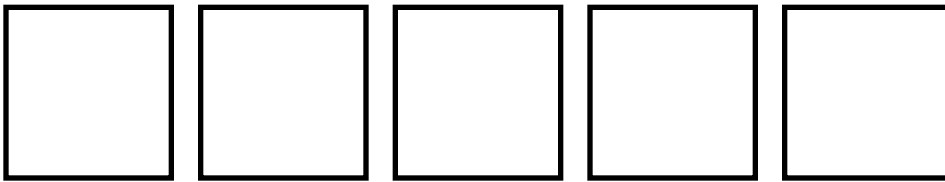
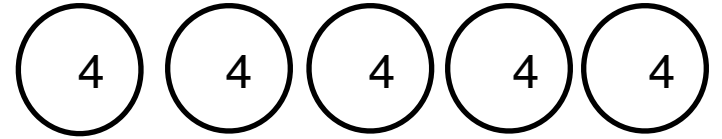
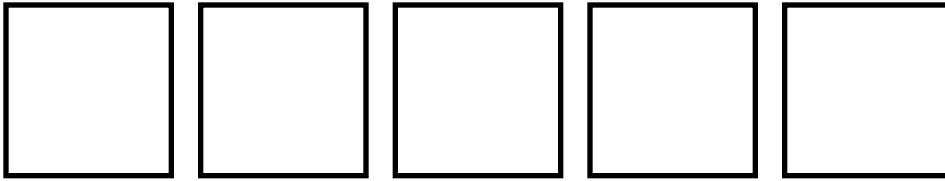
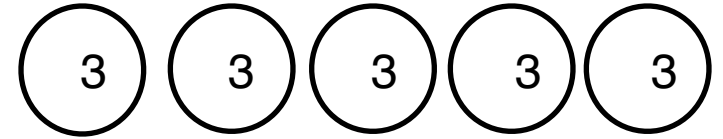
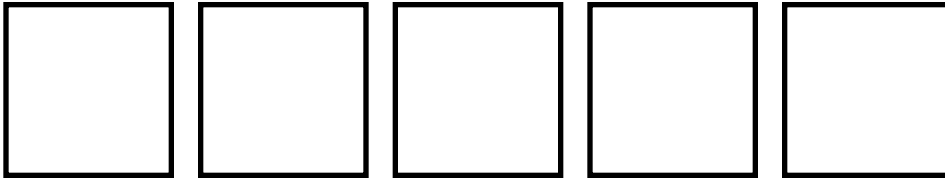
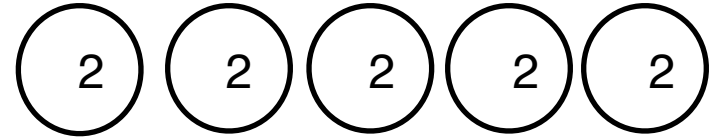
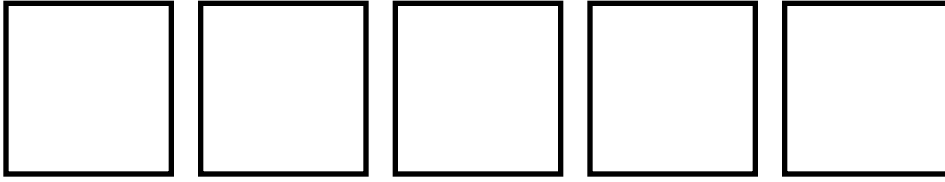
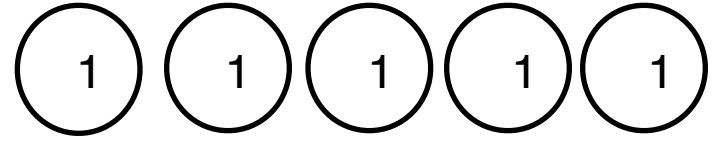
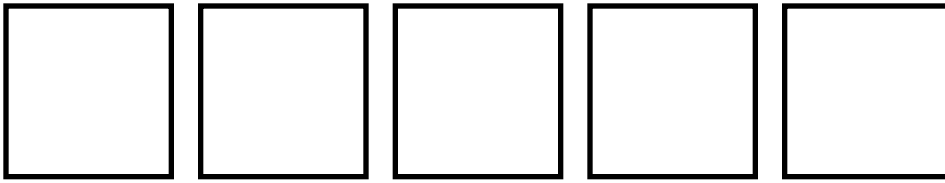


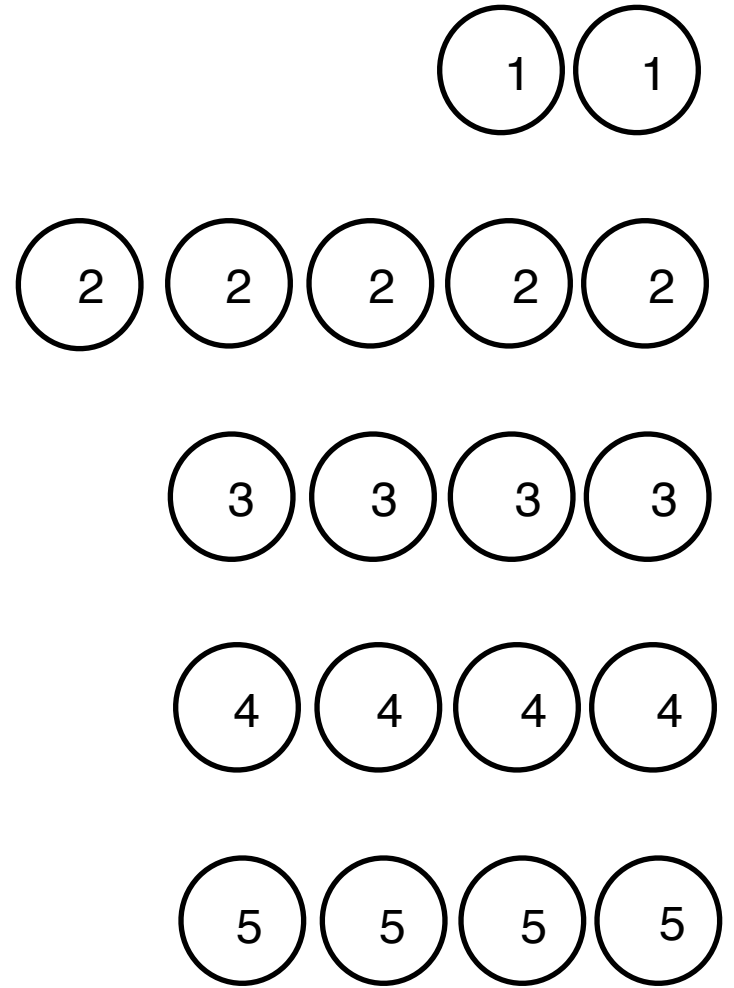
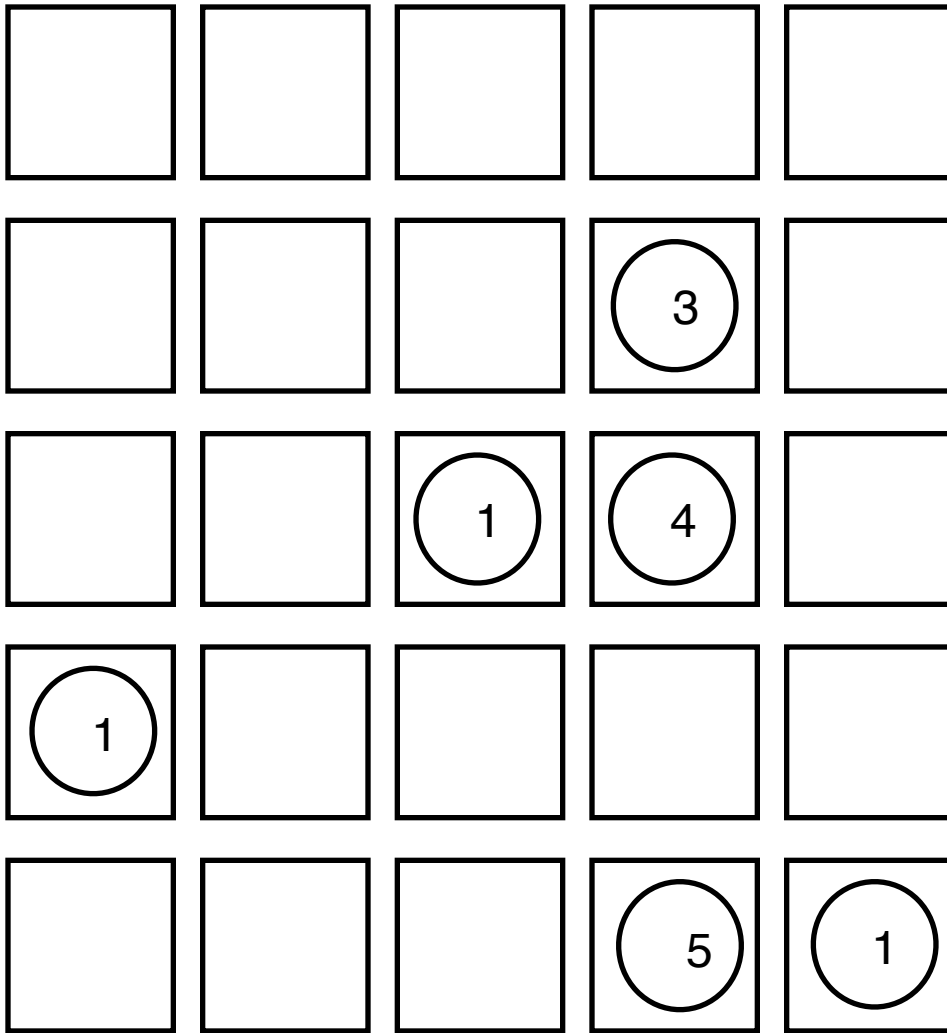
Sudoku

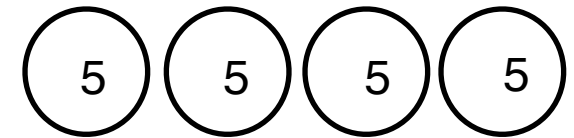
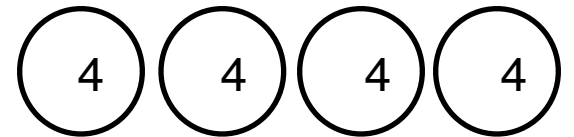
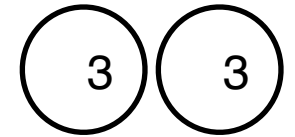
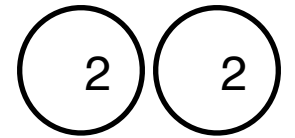
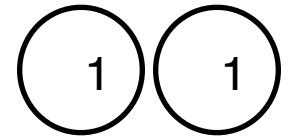
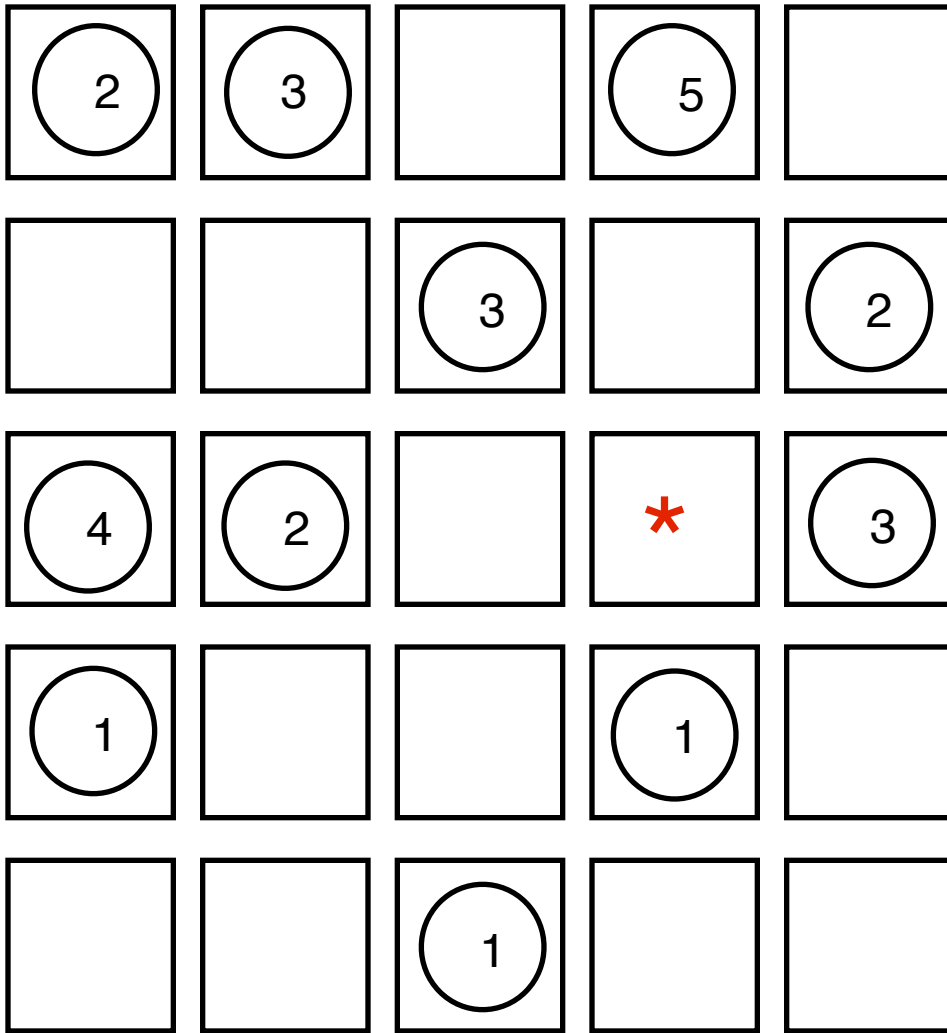
8	3	5	4	1	6	9	2	7
2	9	6	8	5	7	4	3	1
4	1	7	2	9	3	6	5	8
5	6	9	1	3	4	7	8	2
1	2	3	6	7	8	5	4	9
7	4	8	5	2	9	1	6	3
6	5	2	7	8	1	3	9	4
9	8	1	3	4	5	2	7	6
3	7	4	9	6	2	8	1	5

Latin Square as a game?

- Latin square rules
- Last player wins (whoever fails to have a legal move loses)
- Let's try a 5x5 board...

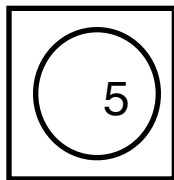
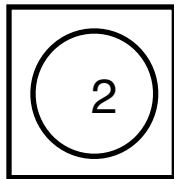
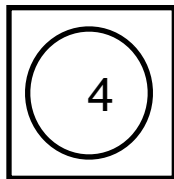
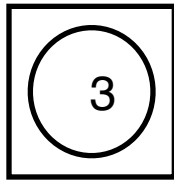
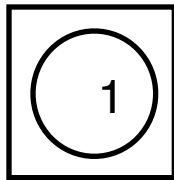






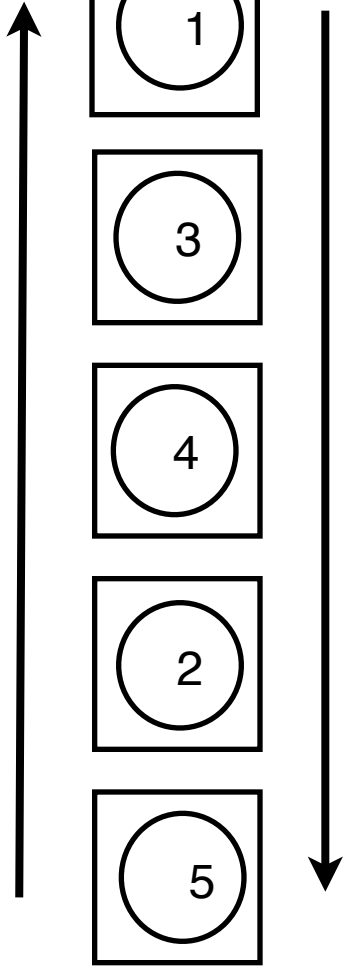
3	4	5	1	2
2	1	4	3	5
5	2	1	4	3
1	5	3	2	4
4	3	2	5	1

Boring...



A

A sequence of five different numbers...

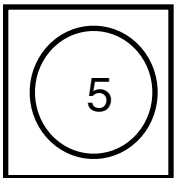
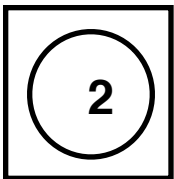
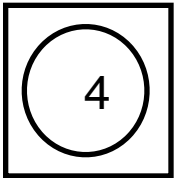
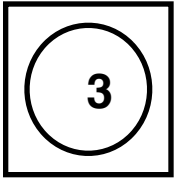
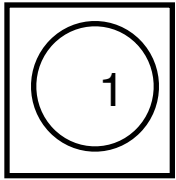


Which player should “own” this column?

B

A

A sequence of five different numbers...

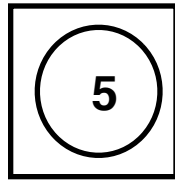
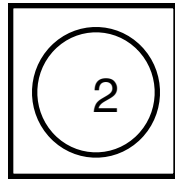
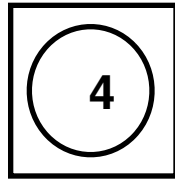
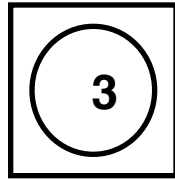
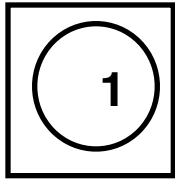


B does not have any increasing subsequence with more than two elements...

B

A

A sequence of five different numbers...



A has an increasing subsequence of length four...

It should be A's column.

B

A Theorem (Erdős e Szekeres, 1935)

In a sequence with $n^2 + 1$ elements there is always a monotone subsequence of length $n + 1$

Our case is $n = 2, n^2 + 1 = 5$

Therefore, each filled column has a monotone 3-sequence

“Proof:”

Let the original sequence be x_1, \dots, x_{n^2+1} .

Let us organize these numbers in columns in the following way:

a) x_1 is the base of the first column;

b) If x_i is larger than the top of some column, then x_i should be placed on top of the first such column; otherwise x_i will be the base of a new column.

Example

3 5 2 4 1

3 5 2 4 1

Example

5 2 4 1

3 5 2 4 1

3

Example

2 4 1

3 5 2 4 1

5
3

Example

4 1

3 5 2 4 1

5
3 2

Example

1

3 5 2 4 1

5 4

3 2

Example

3 5 2 4 1

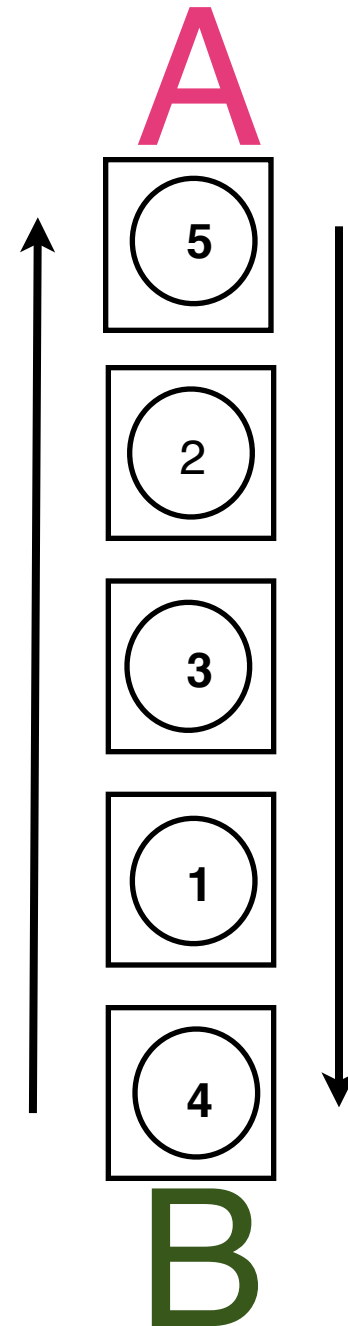
5 4
3 2 1

Whoever gets an increasing
3-sequence in a column wins
that column! Right?

Wrong!

A: 2-3-4

B: 1-3-5



The winner of the column is the player that first gets an increasing 3-sequence!

If they both get their 3-sequences at the same time, the owner of the column is the player moving.

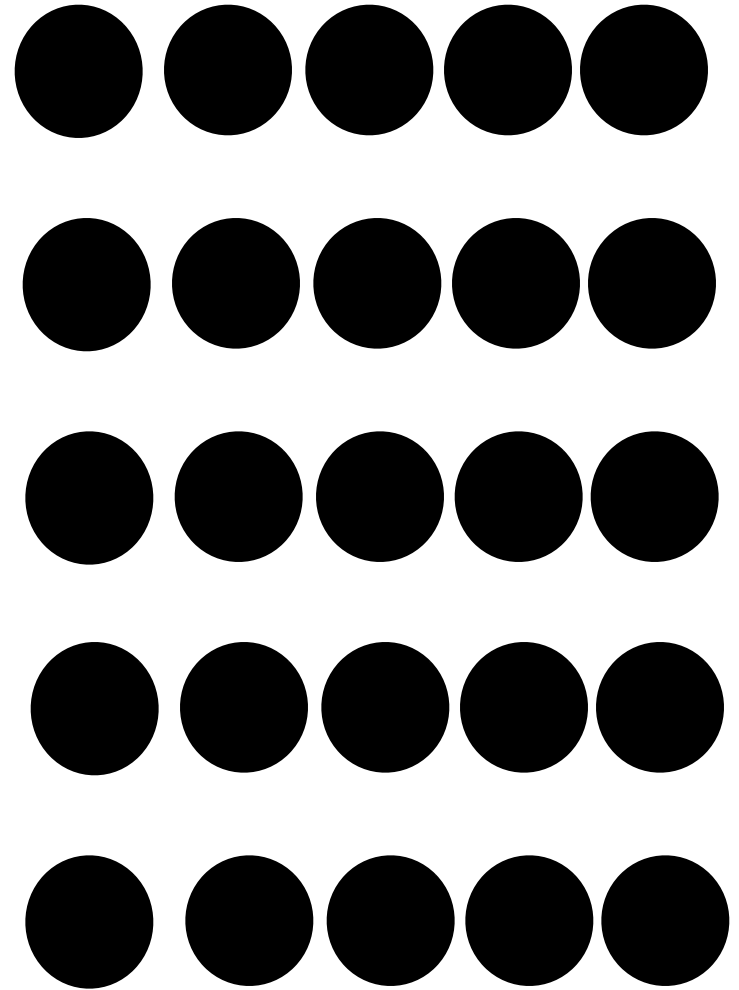
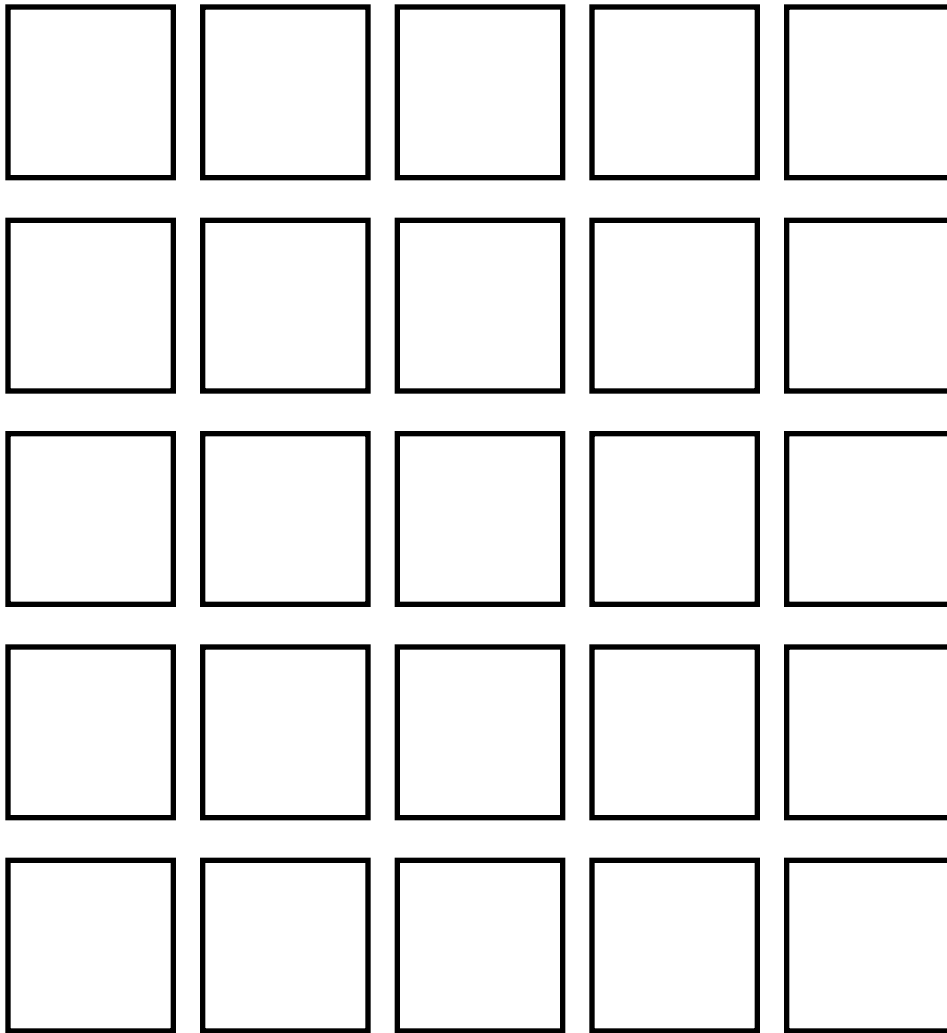
Now we have a game for two players
with no ties!

There are three ways of implementing it:

1 - With a huge chance factor

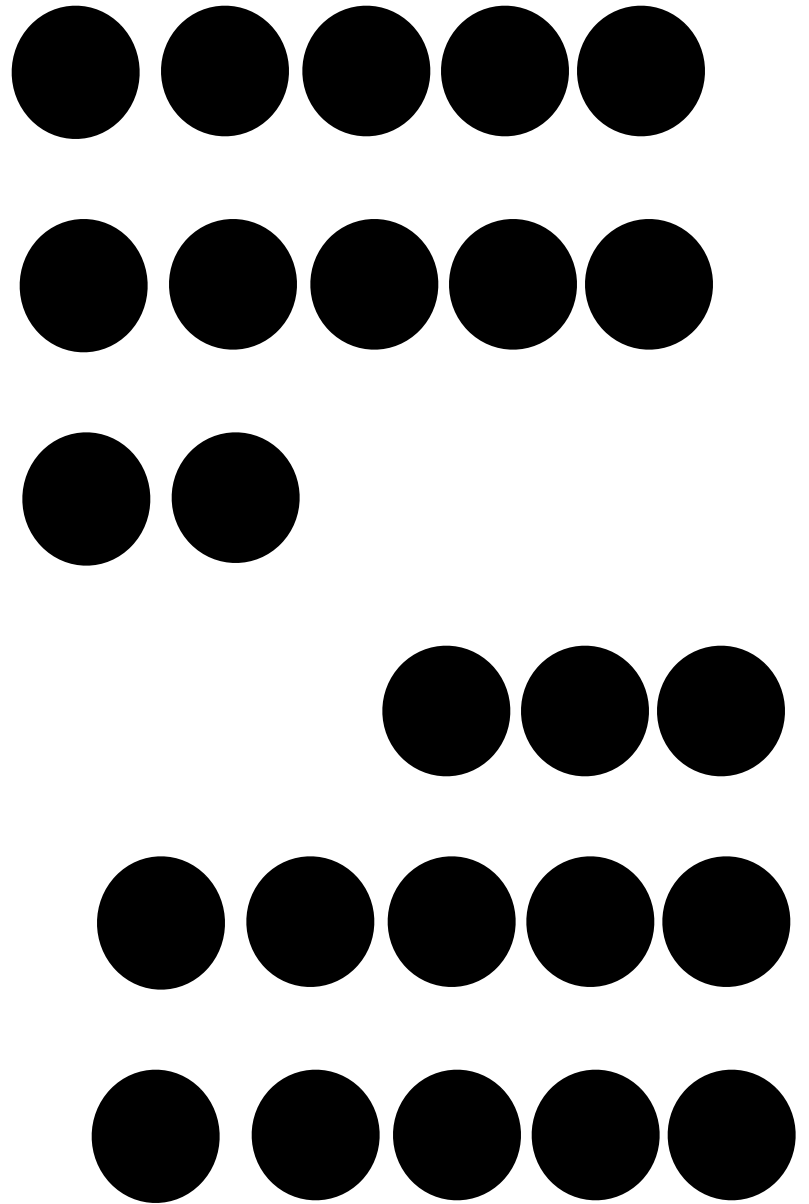
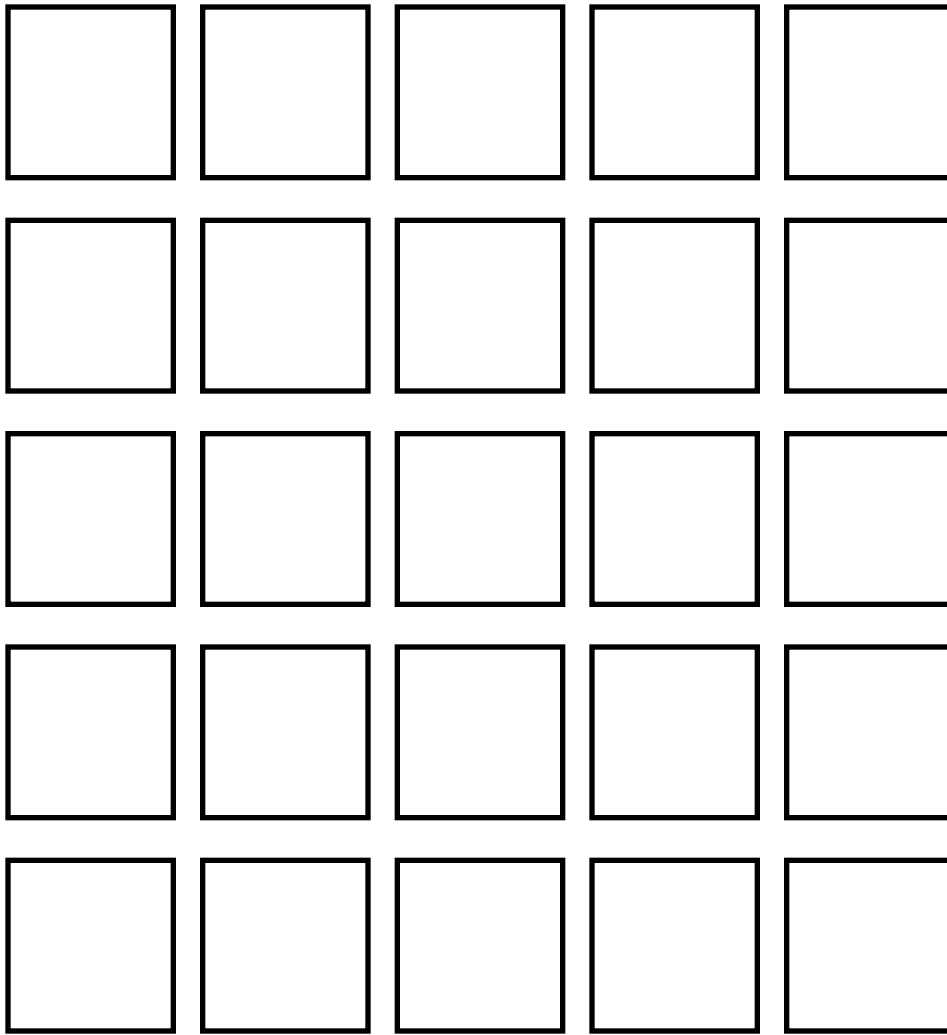
2 - With a medium chance factor

3 - Without any chance factor



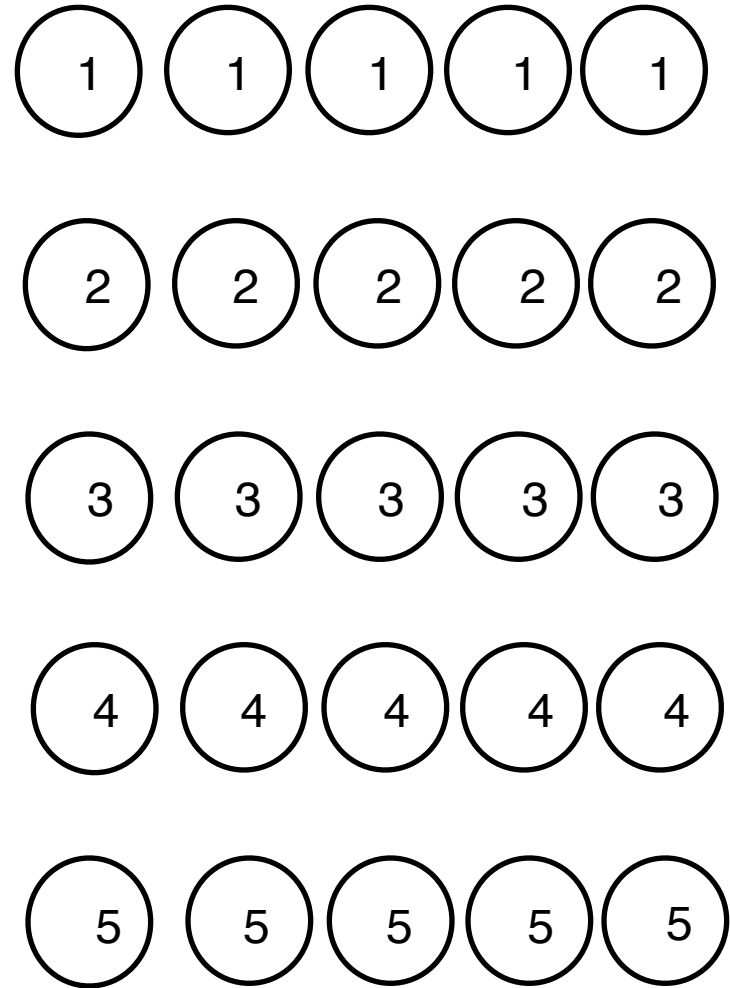
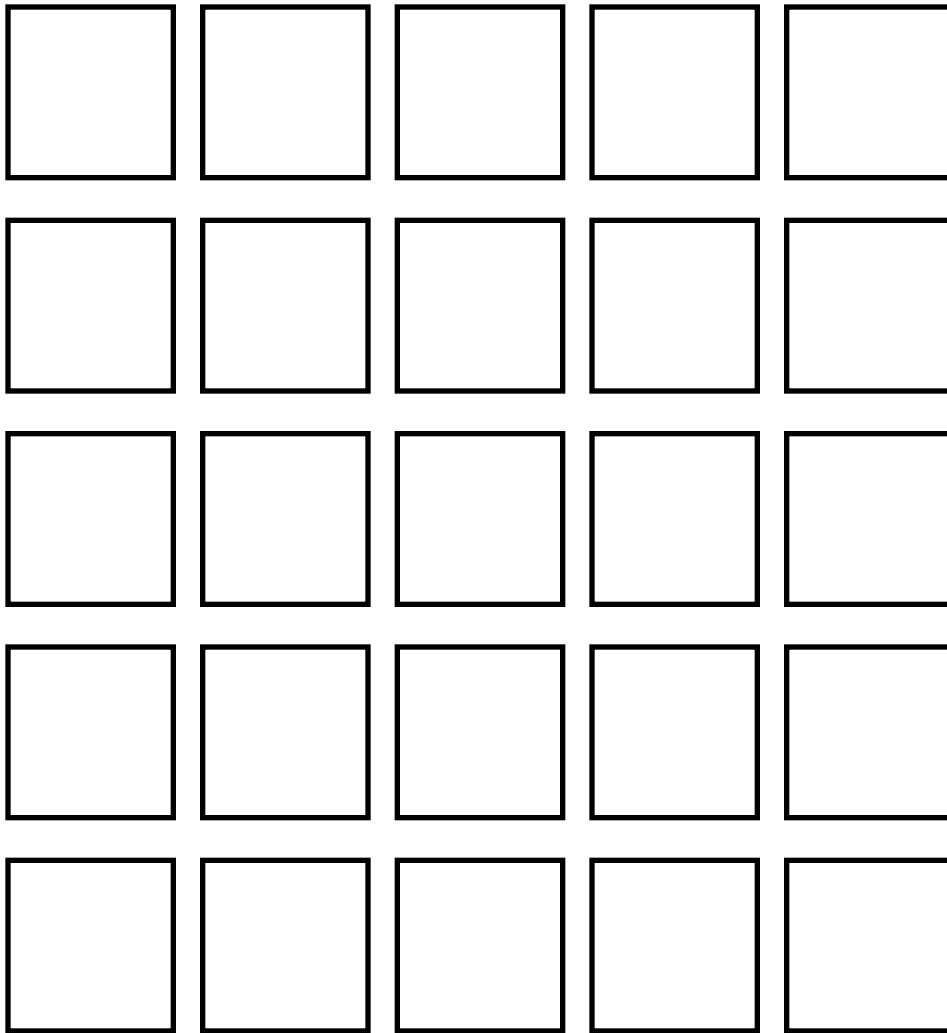
1 - With a huge chance factor

All pieces are turned and shuffled. Pick one at random and place it, if possible.



2 - With a medium chance factor

All pieces are turned. Shuffle and divide them 12-13 before starting.



3 - Without any chance factor

The pieces are visible at all times. Players choose freely the piece to place.

<http://www.luduscience.pt/erdos.html>



available for
IOS and Android
as
Erdos Latino

Thank you!